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*Sean C. Ahern and John W. Norbury*  
*University of Wisconsin-Milwaukee, Milwaukee, Wisconsin*

*R. K. Tripathi*  
*Langley Research Center, Hampton, Virginia*

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*Sean C. Ahern and John W. Norbury*  
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National Aeronautics and  
Space Administration

Langley Research Center  
Hampton, Virginia 23681-2199

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## Nomenclature

Note: the units used in this work are such that  $\hbar = c = 1$

|                                    |  |
|------------------------------------|--|
| $A^\mu$                            | 4-potential  |
| $\vec{A}$                          | vector potential   |
| $\alpha$                           | fine structure constant  |
| $\vec{B}$                          | magnetic field vector  |
| $b$                                | impact parameter in a collision  |
| $b_{\min}$                         | minimum impact parameter effective in a collision  |
| $c$                                | speed of light   |
| $\frac{d^2 I}{dAd\omega}$          | frequency spectrum (of bosons)   |
| $\frac{d\chi}{d\omega}$            | radiation cross section  |
| $\frac{dE_{\text{brem}}}{dX}$      | stopping power (bremsstrahlung)  |
| $\frac{dE_{\text{coll}}}{dX}$      | stopping power (collisional)   |
| $\frac{dE_{\text{radiation}}}{dX}$ | stopping power (general)   |
| $\frac{d\sigma}{d\Omega}$          | scattering cross section   |
| $\Delta E$                         | energy transferred by a boson in a collision   |
| $\Delta p_\perp$                   | a constant that appears in various formulas, equal to $\sqrt{M^2 + (\Delta p_{\perp \min})^2}$                       |
| $\Delta p_{\perp \min}$            | a constant that appears in various formulas, equal to $\frac{\omega}{\gamma v}$                                      |
| $\Delta \vec{p}$                   | momentum transferred by a boson in a collision   |
| $\delta(x)$                        | Dirac delta function   |
| $E$                                | amount of energy transferred to an electron in an atom by an incident particle                                       |
| $E_f$                              | energy of the particle that emits a mediating boson of interest  |
| $e$                                | electromagnetic coupling constant  |
| EM                                 | electromagnetic  |
| $E_0$                              | initial energy of a cosmic ray as it enters a medium of interest   |
| $\varepsilon$                      | a charge parameter   |
| $\eta$                             | a charge parameter   |
| $\vec{E}$                          | electric field vector  |
| $E(X)$                             | energy of a cosmic ray (as a function of depth within a medium of interest)  |
| $\gamma$                           | Lorentz factor   |
| $g_W$                              | charged weak coupling constant   |
| $g_Z$                              | neutral weak coupling constant   |
| $h$                                | Planck's constant  |
| $\hbar$                            | $\frac{h}{2\pi}$   |
| $I_0$                              | a constant that appears in the boson frequency spectrum, equal to $\frac{2}{(2\pi)^3} \frac{q_V^2 + q_A^2}{b^2 v^2}$ |

|                       |   |
|-----------------------|---|
| $J^\mu$               | 4-current   |
| $\vec{J}$             | current density   |
| $K_0(x), K_1(x)$      | modified Bessel functions of the second kind, of orders 0 and 1, respectively                   |
| L                     | longitudinal  |
| $\lambda$             | normalized helicity   |
| $M$                   | mass of a boson that mediates an interaction of interest  |
| $M_{\text{electron}}$ | mass of an electron   |
| $M_f$                 | mass of the particle that emits a mediating boson of interest                                   |
| $N$                   | number density of atoms in a target material  |
| $\omega$              | angular frequency of a boson  |
| $\omega_{\text{max}}$ | maximum angular frequency of a mediating boson  |
| $\Phi$                | scalar potential  |
| $\phi$                | a constant appearing in $\frac{dE_{\text{coll}}}{dX}$ , equal to $b_{\text{min}}\Delta p_\perp$ |
| $p^\mu$               | 4-momentum of boson   |
| $Q$                   | an effective charge, equal to $\sqrt{q_V^2 + q_A^2}$  |
| $q_A$                 | axial-vector charges  |
| $q_{\text{electron}}$ | the charge Q of an electron   |
| $Q^{\text{EM}}$       | dimensionless charge quantum number for electromagnetic interactions                            |
| $q_{\text{incident}}$ | the charge Q of an incident particle  |
| $q^\mu$               | 4-charge  |
| $q_{\text{target}}$   | the charge Q of a target particle   |
| $q_V$                 | vector charge   |
| $\vec{r}$             | position vector   |
| RCS                   | radiation cross section   |
| $\rho$                | charge density  |
| SCS                   | scattering cross section  |
| $\hat{s}$             | spin direction  |
| $s^\mu$               | normalized 4-spin   |
| SP                    | stopping power  |
| $\vec{S}$             | Poynting vector   |
| T                     | transverse  |
| t                     | time  |
| $T^3$                 | third component of the vector of weak isospin quantum numbers                                   |
| $\theta$              | scattering angle of a pulse of bremsstrahlung radiation emitted                                 |
| $\theta_W$            | weak-mixing angle   |
| TSCS                  | Thomson scattering cross section  |
| $u^\mu$               | 4-velocity  |
| $V$                   | velocity of a pulse of bremsstrahlung radiation emitted   |
| $v$                   | magnitude of velocity vector  |
| $\vec{v}$             | velocity vector   |
| $X$                   | distance (within a medium of interest) traversed by cosmic ray                                  |

|           |   |
|-----------|---|
| $X_0$     | radiation length  |
| $x$       | Feynman scaling variable  |
| $\xi$     | a constant that appears in various Bessel functions, equal to $b\Delta p_\perp$ |
| $\hat{z}$ | unit vector pointing in the $+z$ direction                                      |

## Abstract

*Interactions between galactic cosmic rays and matter are a primary focus of the NASA radiation problem. The electromagnetic forces involved are for the most part well documented. Building on previous research, this study investigated the relative importance of the weak forces that occur when a cosmic ray impinges on different types of materials. For the familiar electromagnetic case, it is known that energy lost in the form of radiation is more significant than that lost via contact collisions; the rate at which the energy is lost is also well understood. Similar results were derived for the weak force case. It was found that radiation is also the dominant mode of energy loss in weak force interactions, and that weak force effects are indeed relatively weak compared with electromagnetic effects.*

## 1. Introduction

When a fast-moving charged particle passes through matter, it loses energy and slows down. There are two ways it can lose energy. It can either lose kinetic energy during collisions with other particles or lose energy in the form of radiation. During any given collision, the incident particle will transfer some of its kinetic energy to other particles and thereby undergo an acceleration. It is known that whenever a charged particle is accelerated, it radiates electromagnetic (EM) energy. This type of radiation is referred to as bremsstrahlung (braking radiation) because it was first observed in experiments where high-energy electrons were stopped in a thick metallic target. The theory of energy loss by moving charges is well understood for EM interactions. One important result from these studies is that for nonrelativistic particles, the energy lost in the form of bremsstrahlung is negligible compared with the kinetic energy loss, whereas for relativistic particles, bremsstrahlung can be the dominant form of energy loss. This paper addresses the physics of energy loss by high-energy cosmic rays as they travel through bulk matter, such as the wall of a spacecraft or the Earth's atmosphere. Thus, the bremsstrahlung processes were of central concern.

For most macroscopic particle phenomena, such as the passage of a high-energy particle through bulk matter, energy transfer via EM processes is usually the most significant channel, but contributions from weak force processes can sometimes play the dominant role. The purpose of this project is to generalize the analysis of energy loss by charged particles via EM interactions to that via weak interactions. In particular, one quantity was sought after for both bremsstrahlung and collisional energy loss mechanisms—stopping power (SP),  $dE_{\text{radiation}}/dX$ . The SP is the energy lost by the incident particle per unit path length traversed. The SP generally depends on the energy, mass and charge of the incident particle, and the density and charge of the target material.



## 2. Method

### 2.1. Previous research

The method used here to find the SP via weak interactions for the two types of energy-loss mechanisms was to follow the well-documented formalism for similar EM processes. The reason for this approach was that the equations for weak interactions differ from those for EM interactions by only two parameters: the charge  $Q$  of the particle undergoing the acceleration and the mass  $M$  of the particle mediating the force. Formulas were appropriately generalized to cases where  $Q$  and  $M$  differed from the EM case.

For EM interactions,  $Q$  is the familiar electric charge, which is always an integral multiple of the charge  $-e$  of the electron. For weak interactions,  $Q$  is known as the weak charge and is only slightly more complicated than the electric charge. In all cases,  $Q$  is deduced from a knowledge of the 4-current  $J^\mu = (\rho, \vec{J})$  that describes the charge  $\rho$  and current  $\vec{J}$  densities of the particle in question. The correct form for  $J^\mu$  for any given interaction is provided by the Standard Model. For all electroweak interactions of interest, the 4-current of a point charge in motion is found to be of the form  $J^\mu(\vec{r}, t) = \delta[\vec{r}(t)]q^\mu$ , where  $\delta[\vec{r}(t)]$  is the usual Dirac delta function, which vanishes everywhere except at  $\vec{r}(t) = \vec{0}$ , and  $q^\mu$  is a new quantity called the 4-charge. Generally  $q^\mu$  is found to be of the form  $q^\mu = q_V u^\mu + q_A s^\mu$ , where  $u^\mu$  and  $s^\mu$  are the 4-velocity and normalized 4-spin, respectively, of the particle and  $q_V$  and  $q_A$  are coefficients (the vector and axial-vector charges, respectively) that depend on various well-known parameters (viz, electric charge, weak isospin and weak mixing angle) that appear in the Standard Model. In a frame in which the particle's velocity is  $\vec{v} = v\hat{z}$  and spin direction is  $\hat{s}$ ,  $u^\mu = \gamma(1, \vec{v})$  and  $s^\mu = \gamma(\lambda v, \hat{s})$ , where  $\gamma = 1/\sqrt{1-v^2}$  is the Lorentz factor and  $\lambda = \hat{s} \cdot \hat{z}$  is the normalized helicity of the particle. Note that if the direction of “spin-up” is taken to be  $+\hat{z}$ , then  $\lambda$  will be either  $+1$  or  $-1$ , and, further, in the ultrarelativistic limit of interest (where  $v \rightarrow 1$ ),  $s^\mu = \lambda u^\mu$ , and thus  $J^\mu(\vec{r}, t) = \delta[\vec{r}(t)](q_V + \lambda q_A)u^\mu$ . Using this expression for the 4-current, a formula for the 4-potential  $A^\mu = (\Phi, \vec{A})$ , which describes the scalar  $\Phi$  and vector  $\vec{A}$  potentials of the point charge, was deduced (as a solution to the Proca equation). Without specifying the boson mass  $M$  at this point,  $A^\mu$  is related to  $J^\mu$  by the following equation:

$$A^\mu(\vec{r}, t) = \frac{1}{4\pi} \int d^3\vec{r}' \frac{J^\mu(\vec{r}', t)}{|\vec{r} - \vec{r}'|} e^{-M|\vec{r} - \vec{r}'|}. \quad (1)$$

The primed quantities in this equation label the (differential) source charge elements. By using the expression for  $J^\mu$  involving the Dirac delta function, the integral is easily seen to reduce to the 4-vector expression

$$A^\mu(\vec{r}, t) = \frac{1}{4\pi} \frac{q^\mu}{|\vec{r}(t)|} e^{-M|\vec{r}(t)|}. \quad (2)$$

Electric  $\vec{E}$  and magnetic  $\vec{B}$  fields are found from these equations in the same way that they are in

the massless case:

$$\vec{E}(\vec{r}, t) = -\vec{\nabla}\Phi(\vec{r}, t) - \frac{\partial\vec{A}(\vec{r}, t)}{\partial t} \quad (3a)$$

$$\vec{B}(\vec{r}, t) = \vec{\nabla} \times \vec{A}(\vec{r}, t). \quad (3b)$$

For a point charge moving at  $v \simeq 1$  in the  $\hat{z}$  direction, the only nonvanishing components of the potentials and fields, evaluated at impact parameter  $b$ , are

$$\Phi(b, t) = \frac{1}{4\pi} \frac{\gamma(q_V + \lambda q_A)}{r} e^{-Mr} \quad (4a)$$

$$A_z(b, t) = \frac{1}{4\pi} \frac{\gamma(q_V + \lambda q_A)}{r} e^{-Mr} \quad (4b)$$

$$E_x(b, t) = \frac{1}{4\pi} \frac{\gamma(q_V + \lambda q_A)b}{r^3} (1 + Mr) e^{-Mr} \quad (4c)$$

$$E_z(b, t) = -\frac{1}{4\pi} \frac{\gamma q_V v t}{r^3} (1 + Mr) e^{-Mr} \quad (4d)$$

$$B_y(b, t) = \frac{1}{4\pi} \frac{\gamma(q_V + \lambda q_A)b}{r^3} (1 + Mr) e^{-Mr}, \quad (4e)$$

where  $r = \sqrt{b^2 + (\gamma vt)^2}$ . The quantity of central concern to energy transport by the fields is the Poynting vector,  $\vec{S}$ . For massive fields traveling through a vacuum,  $\vec{S}$  can be shown to be generally of the form  $\vec{S} = \vec{E} \times \vec{B} + M^2 \Phi \vec{A}$ . Clearly it will always depend quadratically on the charges. Furthermore, as seen above, the helicity  $\lambda$  of the point charge invariably enters into this factor. As the usual application of any such theory is to unpolarized beams of particles, an average overall possible helicity state of the charge is of interest. It turns out that the charge-related quantity that appears in the final set of equations describing the distribution of bosons outside the moving charge is the quantity  $q_V^2 + q_A^2$ .  $Q$  is thus defined as  $Q = \sqrt{q_V^2 + q_A^2}$ .  $Q$  is not the usual canonical charge (which happens to be simply  $q_V$ ), as defined via the Noether prescription, so it can only be referred to as some sort of (Lorentz invariant) effective charge. For electromagnetic interactions,  $q_V = Q^{\text{EM}} e$  and  $q_A = 0$ , where  $e = \sqrt{4\pi\alpha}$  is the electromagnetic coupling constant ( $\alpha \simeq 1/137$  is the fine structure constant) and  $Q^{\text{EM}}$  is the familiar dimensionless charge quantum number for electromagnetic interactions (e.g.,  $Q^{\text{EM}} = \pm 1$  for positrons and electrons, respectively). For neutral weak interactions (mediated by Z bosons),  $q_V = g_Z(T^3 - 2Q^{\text{EM}} \sin^2 \theta_W)/2$  and  $q_A = -g_Z T^3/2$ , where  $\theta_W \simeq 28.74^\circ$  is the weak-mixing angle,  $T^3$  is the third component of the vector of weak isospin quantum numbers of the point charge (e.g.,  $T^3 = \pm 1/2$  for neutrinos and electrons, respectively), and  $g_Z = e/\sin \theta_W \cos \theta_W$  is the neutral weak coupling constant. And finally, for charged weak interactions (mediated by  $W^\pm$  bosons),  $q_V = g_W/2\sqrt{2}$  and  $q_A = \mp g_W/2\sqrt{2}$ , respectively, where  $g_W = e/\sin \theta_W$  is the charged weak coupling constant.

One guiding principle for determining  $M$  was conservation of energy and momentum. So  $M^2 = (\Delta E)^2 - (\Delta \vec{p})^2$ , where  $\Delta E$  and  $\Delta \vec{p}$  are the energy and momentum transferred (mediated by the

boson) in the collision, serves as a defining equation for this parameter. The boson masses are not uniquely defined in general by this equation, however, and an additional piece of information is needed. The Lorentz condition in momentum-space,  $p_\mu A^\mu = 0$ , yields a relation between  $\Delta E$  and  $\Delta \vec{p}$  (here  $p^\mu = (\Delta E, \Delta \vec{p})$  is the boson's 4-momentum) that can be used to eliminate this ambiguity. It was actually one of the more interesting results of this study that  $\Delta E$  and  $\Delta \vec{p}_z$  (the z-component of  $\Delta \vec{p}$ ) are found to be related to one another by way of the above-mentioned charges:  $\Delta |\vec{p}_z| = \Delta E(1 - \varepsilon/\gamma^2)/v$ , where  $\varepsilon \equiv q_A \lambda / (q_V v + \lambda q_A)$  is a new charge parameter. After taking the ultrarelativistic limit, averaging overall possible helicity states of the colliding particle, and making other slight reparameterizations that ensure  $M$  is Lorentz invariant and the concept of causality is preserved (i.e., the boson travels subluminally), an equation for  $M$  was arrived at that is unique to each of the three types of interactions of interest. The final mass scheme is as follows:

$$M = M_f \sqrt{\eta \left(\frac{x}{v}\right) \left[1 - \left(\frac{x}{v}\right)\right]}. \quad (5)$$

A new charge parameter,  $\eta$ , appears here. It is very much like  $\varepsilon$  defined above, and, in fact, is derived from  $\varepsilon$ . For the photon,  $\eta$  is found to be  $\eta = q_A^2 / (q_A^2 - q_V^2) = 0$ , so that  $M = 0$ , as one might expect. For the Z boson,  $\eta = q_A / (q_A - q_V) = T^3(T^3 - Q^{\text{EM}} \sin^2 \theta_W)/2$ , and  $\eta = 2q_A / (q_A \mp q_V) = 1$  for the  $W^\pm$  boson, respectively. Interestingly, the masses of the W and Z bosons do not equal the masses predicted by the Standard Model (80.419 and 91.1882 GeV, respectively), and in fact can be orders of magnitude different, yet the distribution functions for the bosons using this mass scheme agree exactly with the well-known distribution functions found in the parton theory for electroweak interactions. It is also noteworthy to point out that the ratio of the masses of the Z and W bosons here is roughly the same as the ratio found in the Standard Model. For example, if the colliding particles are either protons or electrons, this ratio is 0.9300 here, and  $\cos \theta_W = 0.8768$  in the Standard Model. The other parameters found in the above equation are as follows.  $M_f$  is the mass of the particle ( $f$  for fermion) that emits the boson and  $x = \Delta E / E_f$  ( $E_f$  is the energy of the particle) is the Feynman scaling variable, which is bounded (by conservation of energy and causality) by the inequality string  $\eta v / [(\gamma v)^2 + \eta] \leq x \leq 1 - 1/\gamma$ . In summary, a boson mass scheme was derived from the basic concepts of conservation of energy and momentum, causality, and Lorentz invariance that uniquely specifies the masses of the equivalent bosons in the theory developed here. Despite the fact that the boson masses here are not constant and the values of the masses of the W and Z bosons can be quite different than those predicted by the Standard Model, the boson distribution functions are in perfect agreement with those in the parton theory for electroweak interactions.

## 2.2. Bremsstrahlung Energy Loss

The calculation of the SP for bremsstrahlung processes involved the determination of several intermediate quantities. The most difficult one to derive was the frequency spectrum,  $d^2 I / dA d\omega$ , of the field of bosons outside the charge; this quantity was referred to above as the boson distribution function.  $d^2 I / dA d\omega$  represents the total amount of energy carried by the field per unit boson

energy per area element on the wavefront of the Lorentz-contracted  $\vec{E}$  and  $\vec{B}$  fields. In short, it is the Fourier transform of the Poynting vector  $\vec{S} = \vec{E} \times \vec{B} + M^2 \Phi \vec{A}$  mentioned above.

A slight subtlety had to be addressed in deriving  $d^2I/dAd\omega$ . EM radiation is known to be polarized in a plane perpendicular to the direction of propagation, which means that when an EM wave (i.e., a swarm of photons) impinges on a charged particle, the particle will only move in that transverse plane. But radiation associated with massive bosons can exist in an additional state—longitudinal polarization, which means that the induced motion can take place in practically any direction relative to the direction of propagation of the wave. So there are actually two different frequency spectrum functions—one corresponding to transversely-polarized bosons (designated with a T) and the other describing longitudinally-polarized bosons (designated with an L); it can be shown that only the T boson states contribute to the bremsstrahlung process. The frequency spectrum for T boson states is

$$\left( \frac{d^2I}{d\omega dA} \right)_T = I_0 \xi^2 K_1^2(\xi), \quad (6)$$

where

$$I_0 \equiv \frac{2}{(2\pi)^3} \frac{q_V^2 + q_A^2}{b^2 v^2} = \text{const} \quad (7)$$

and

$$\xi \equiv b \sqrt{M^2 + \left( \frac{\omega}{\gamma v} \right)^2}. \quad (8)$$

$K_1$  is a modified Bessel function of the second kind, of order 1,  $b$  is the impact parameter in the collision, and  $\omega = \Delta E$  is the angular frequency of the boson.

Next, the radiation cross section (RCS),  $d\chi/d\omega$ , which conveys the probability for a boson with a specified energy to be emitted from one charge and subsequently absorbed by another, was determined. It is, in a crude sense, the product of  $d^2I/dAd\omega$  and another cross section, the scattering cross section (SCS),  $d\sigma/d\Omega$ , all integrated over a solid angle of  $4\pi$  steradians and the (infinite) area of the wavefront of Lorentz contracted  $\vec{E}$  and  $\vec{B}$  fields. The SCS can be thought of as the time average of the probability for the incident charge to be deflected into a certain specified direction during the collision with another charge. For the EM case, this quantity is referred to as the Thomson scattering cross section (TSCS). A generalized version of it was derived here from first principles:

$$\frac{d\sigma}{d\omega} = \frac{(q_{\text{incident}})^2 (q_{\text{target}})^2}{M_f^2 V^2} \frac{1}{2} (1 + \cos^2 \theta) e^{-2Mb}. \quad (9)$$

$(q_{\text{incident}})^2$  is  $Q^2$  for the incident particle (e.g., the incident cosmic ray),  $(q_{\text{target}})^2$  is  $Q^2$  for the target particle (e.g., an atom in the spacecraft wall),  $V$  is the velocity of the pulse of bremsstrahlung radiation emitted, and  $\theta$  is the scattering angle of the pulse of bremsstrahlung radiation emitted. The

formula reduces to the familiar Thomson formula when the charges are set to the electric charge  $e$ , and  $M$  is set to zero.

After these two quantities determining  $d\chi/d\omega$  were identified, it was found that, unlike the traditional RCS, the generalized RCS could not be a simple product of the two above-mentioned terms. After ironing out this technicality, a final formula was found for the generalized RCS that immediately yields a generalized version of the SP:

$$\frac{d\chi}{d\omega} = \frac{16}{3} \frac{1}{V^2 v^2} \frac{(q_{\text{incident}})^2 (q_{\text{target}})^2}{M_f^2} \int_{b_{\min}}^{\infty} db \frac{\xi^2}{b} K_1^2(\xi) e^{-2Mb}, \quad (10)$$

where  $b_{\min}$  is the minimum impact parameter effective in the collision. A detailed analysis shows that, for any of the three types of electroweak interactions of interest, the overall behavior of the RCS is dominated by the  $\omega \rightarrow 0$  limit. Given that the energy  $\Delta E$  of the boson that mediates the interaction is related to  $\omega$  via the equation  $\Delta E = \omega$ , it can be said that the nonrelativistic bosons (i.e., those with  $\Delta E \ll E_f$ , or  $x \ll 1$ ) contribute the most to the generation of bremsstrahlung. The boson mass scheme then implies that those bosons with  $M \ll M_f$  make the greatest contribution. As such, the choice of  $b_{\min}$  for the massive mediator cases should be exactly the same as that made for the photon case. It is easily worked out that  $b_{\min} = 1/2M_f v$  is the appropriate choice. The generalized SP cannot be cast into a simple compact formula and necessarily involves a two-fold integral:

$$\frac{dE_{\text{brem}}}{dX} = N \int_0^{\omega_{\max}} d\omega \frac{d\chi}{d\omega}, \quad (11)$$

where  $N$  is the number density of atoms in the target material, and  $\omega_{\max}$  is the maximum angular frequency (allowed by conservation of energy) of the mediating boson.

In contrast to the above equation, the formula for the traditional (massless mediator) SP is a one-dimensional integral. Numerical methods were needed to solve the integrals. Numerical convergence of the integral expression for the RCS was assured by comparing the numerical result with the expected analytic result for the photon-mediated bremsstrahlung process. The same integration step size was then used to calculate the RCS for W- and Z-boson-mediated interactions. As mentioned above, the SP was then determined by performing one more integral, using the same integration step size as before. For ultrarelativistic cosmic rays (in the complete screening limit), the RCS turned out to simply be a constant (independent of the energy of the cosmic ray) for the photon case, and very nearly a constant for the W- and Z-boson cases. As such, the SP is the product of the RCS, the density of the bulk matter, and the energy of the cosmic ray.

Written in another more intuitive way, the energy  $E(X)$  of the cosmic ray can be shown to be related to the distance traversed  $X$  as:  $E(X) = E_0 e^{-X/X_0}$ . Here,  $E_0$  is the energy of the cosmic ray as it enters the material and  $X_0$  is a parameter known as the radiation length. The radiation

length is the distance it takes for the particle's energy to fall to  $1/e$  of its initial value. According to an oversimplified analysis (found in the literature) of EM bremsstrahlung emitted by an electron traversing three different possible materials (lead, aluminum, and air),  $X_0$  is about 0.4 cm for lead, 7 cm for aluminum, and 270 m for air. These results were verified, and corresponding values of  $X_0$  for both W- and Z-boson-mediated processes were determined. For lead,  $X_0$  was found to be 45 m and 7.9 cm for the W- and Z-boson cases, respectively; for aluminum,  $X_0$  was found to be 20 m and 3.3 m for the W- and Z-boson cases, respectively; and for air,  $X_0$  was found to be 20 km and 14 km for the W- and Z-boson cases, respectively. As can be gleaned from the above formula, smaller values of  $X_0$  correspond to greater amounts of energy lost in the form of bremsstrahlung. So clearly, the greatest amount of bremsstrahlung energy is carried away by photons and the least amount is carried away by W-bosons. In fact, the proportions of bremsstrahlung energies carried away by these three particles can be easily determined. If an electron is the incident cosmic ray (traveling at any speed), the photon : Z- boson : W-boson ratios of bremsstrahlung energy loss are roughly  $1 : 10^{-2} : 10^{-5}$  for lead,  $1 : 10^{-2} : 10^{-4}$  for aluminum, and  $1 : 10^{-2} : 10^{-2}$  for air.

The cases considered in this study assume the cosmic ray to be an electron because electrons are the most efficient of all cosmic rays in emitting bremsstrahlung. As a rule of thumb, greater amounts of bremsstrahlung will be produced by cosmic rays with higher charge-to-mass ratios traveling through materials with greater charges. The most efficient scenario turns out to be an electron traveling through a dense material such as lead. In figure 1, a comparison is made between the fraction of cosmic ray energy lost and bremsstrahlung associated with the three electroweak bosons for a 500 GeV electron traveling through a 0.4 cm slab of lead. As alluded to above, the energy lost to W- and Z-boson bremsstrahlung is a mere 0.01 and 4 percent, respectively, of that lost to EM bremsstrahlung. So, it can be reasonably stated that the overwhelming amount of energy lost in the form of bremsstrahlung is carried away by photons.

### 2.3. Collisional Energy Loss

The calculation of the SP for collisional energy loss processes was easier than that for bremsstrahlung processes. Standard Methods (in particular, Fourier analysis) were employed to find the amount of energy  $E$  transferred to an electron orbiting an atom in a grazing collision with a fast-moving (incident) charge,

$$E = \frac{1}{8\pi^2} \frac{(q_{\text{incident}})^2 (q_{\text{electron}})^2}{M_{\text{electron}} v^2 b^2} \left\{ [\xi K_1(\xi)]^2 + \left[ \frac{\Delta p_{\perp \min}}{\gamma \Delta p_{\perp}} \xi K_0(\xi) \right]^2 \right\}, \quad (12)$$

where  $\Delta p_{\perp \min} = \omega/\gamma v$ ,  $\Delta p_{\perp} = \sqrt{M^2 + (\Delta p_{\perp \min})^2}$ , and  $\xi = b \Delta p_{\perp}$ . It was a simple matter to convert this generalized  $E$  into a generalized SP. The SP found for these collisional processes was much simpler in form than that for bremsstrahlung processes; unlike the bremsstrahlung case, the

SP here reduced to a simple compact formula, albeit quite complicated:

$$\begin{aligned} \frac{dE_{\text{coll}}}{dX} = \frac{1}{4\pi} \frac{N(q_{\text{incident}})^2(q_{\text{electron}})^2}{M_{\text{electron}}v^2} & \left\{ \phi K_1(\phi) K_0(\phi) \right. \\ & \left. - \frac{1}{2} \left[ 1 - \left( \frac{\Delta p_{\perp \text{min}}}{\gamma \Delta p_{\perp}} \right)^2 \right] \phi^2 [K_1^2(\phi) - K_0^2(\phi)] \right\}, \end{aligned} \quad (13)$$

where  $\phi = b_{\text{min}} \Delta p_{\perp}$ . When  $Q$  is set to the electric charge and  $M$  is set to zero, the formula reduces to the familiar traditional version.

One of the main goals of this paper is to compare the amount of energy lost via this mechanism with that lost via bremsstrahlung for the three different types of electroweak interaction. It is known that the former energy loss mechanism dominates the latter for slower moving electrons. Again, an electron is taken to be the incident cosmic ray particle because it is the most efficient of all particles at radiating energy. Specifically, it is known that collisional energy loss dominates bremsstrahlung energy loss if the relativistic factor  $\gamma$  (energy-to-mass ratio) of the cosmic ray is less than about 20 if the bulk matter through which the electron travels is lead, and 200 if the material is air. These thresholds (for EM processes) were verified in the present analysis, though the values found were about 17 and 170, respectively, and corresponding ones were found for W- and Z-boson processes. In lead, the threshold value of  $\gamma$  was found to be about  $8.4 \times 10^4$  and 17 for W- and Z-boson processes, respectively. In air, the threshold value of  $\gamma$  was found to be about  $5 \times 10^3$  and  $1 \times 10^3$  for W- and Z-boson processes, respectively. The results, assuming the cosmic ray to be an electron and the target to be made of lead, are shown in figure 2, where the ratio of the SP for the bremsstrahlung process over that for collisional energy loss process is plotted against  $\gamma$ . The threshold above wherein bremsstrahlung dominates collisional energy loss corresponds to 1 along the y-axis. This threshold is clearly surpassed for all three types of electroweak interactions above collision energies corresponding to  $\gamma = 10^5$ . The conclusion to be drawn from these results, then, is the fact that for ultrarelativistic cosmic rays, where  $\gamma$  is typically greater than about  $10^6$ , the dominant energy loss mechanism is most assuredly bremsstrahlung, instead of collisions, for all three types of electroweak interactions.

### 3. Conclusions

In summary, the stopping power for a cosmic ray traveling through bulk matter was derived taking into account both collisional and Bremsstrahlung energy-loss mechanisms. Numerical routines were used to evaluate the stopping power for different scenarios. In all cases, the cosmic ray was assumed to be an electron, and three different possible materials were considered as the bulk matter: lead, aluminum, and air. One major result of the project was that bremsstrahlung dominates the collisional energy-loss mechanism for the ultrarelativistic cosmic rays of interest for all three different types of electroweak interactions. The other major result showed that the bremsstrahlung associated with W- and Z-bosons only enters as a correction at the 1-percent level to electromagnetic bremsstrahlung. This latter result is the more important one for the purposes of figuring out

the influence that the most energetic forms of space radiation have on humans. It shows that weak force effects can be safely ignored compared with similar electromagnetic effects.

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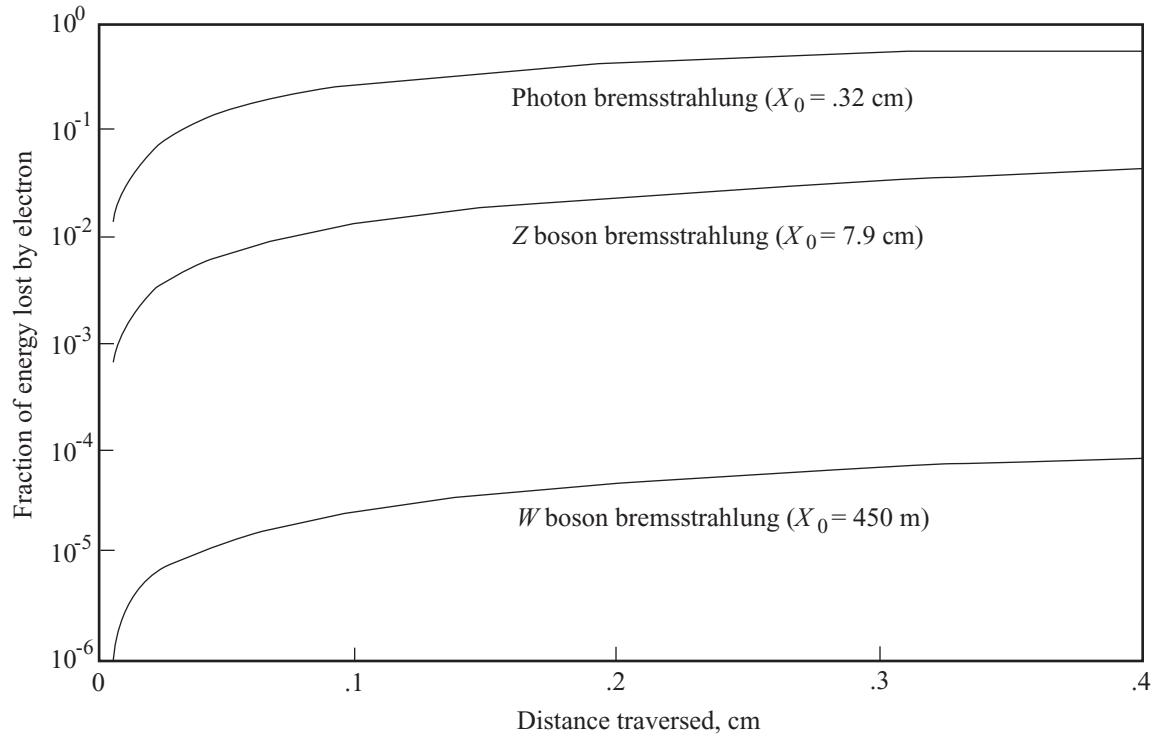


Figure 1. Fraction of energy lost due to Bremsstrahlung by a 500 GeV electron traversing 0.4 cm of lead.

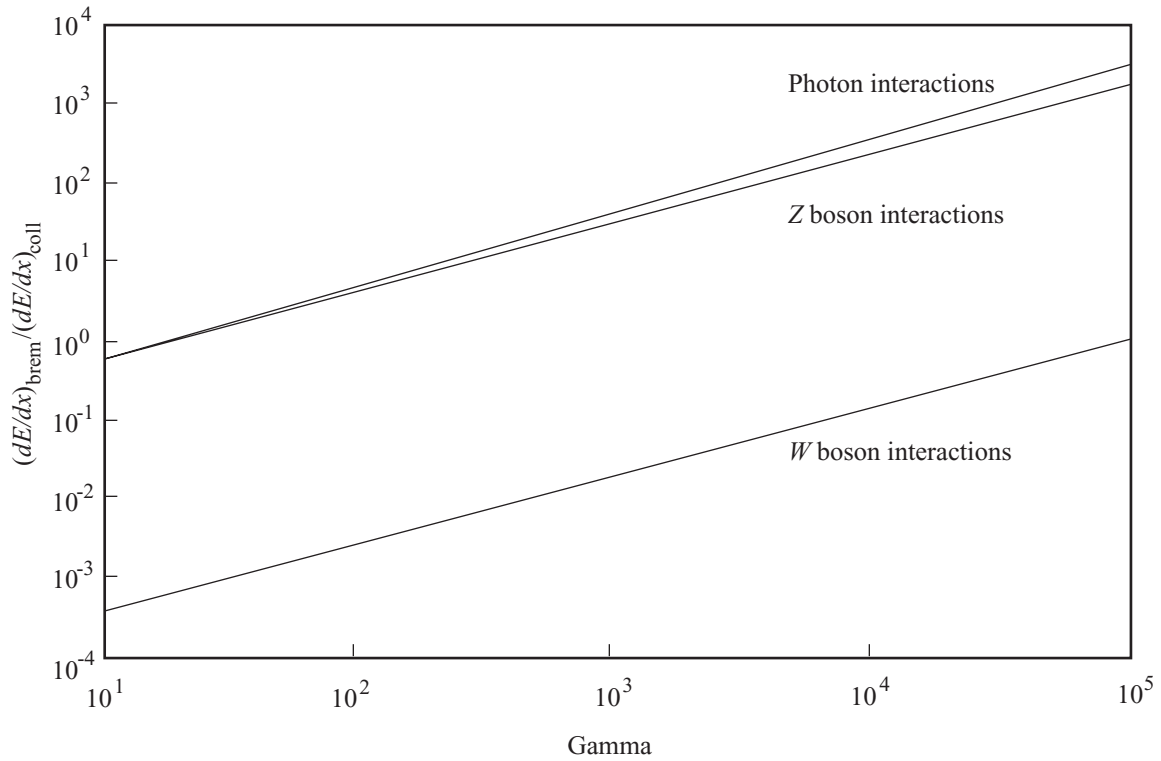


Figure 2. Ratio of SP due to Bremsstrahlung over SP due to collisions versus gamma for an electron traveling through lead.

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